Evaluating students' perceptions of the roles of mathematics in society

Toshikazu Ikeda, Professor of Mathematics Education at Yokohama National University, has found that while mathematical modelling is often evaluated with respect to mathematical attributes, little academic consideration has been given to the non-mathematical viewpoint. To fill this knowledge void, he has developed an analytical tool to evaluate the changes in students' perceptions of the roles of mathematics in society following an experimental teaching program.

Having students recognise the roles that mathematics takes in society is a significant aim in the teaching and learning of mathematical modelling. This issue has been considered by academics since the 1980s. There have been several studies investigating student belief about the value of mathematics in society, but little has changed in the literature regarding the many roles of mathematics in society.

CLASSIFICATION OF MATHEMATICAL MODELS

Mathematical models are often classified according to their various attributes: whether they are concrete or abstract, descriptive or analytical, together with their underlying mathematical basis, such as algebra, geometry or statistics. While useful, these categories are essentially founded on mathematical perceptions. Toshikazu Ikeda, a Professor of Mathematics Education at Yokohama National University, considers the non-mathematical viewpoint in his study and has developed an analytical tool to assess how students perceive the roles of mathematics in society.

METHODOLOGY

The Likert-scales method, where items are usually rated from 'strongly agree' to 'strongly disagree', is often used to assess students' awareness of how useful mathematics is in the real world. While this is a simple and effective tool, it does not capture detailed information regarding students' perceptions of the roles of mathematics in society.

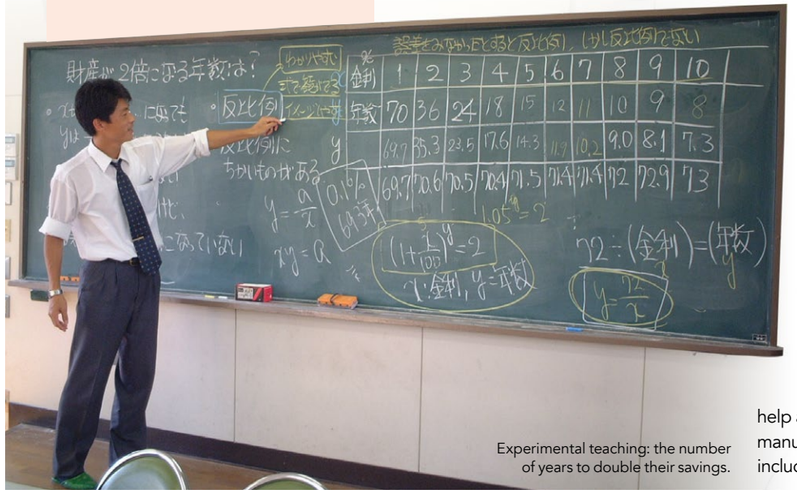
Professor Ikeda has developed four categories which are centred on why mathematical models are used in society. These combine three standpoints: personal-societal perspectives, clarity of role statements, and specific-general contexts.

CATEGORIES

In the first category, students see mathematical modelling only from a personal perspective. For example, they realise the usefulness of carrying out their personal financial calculations, but ignore the impact on society.

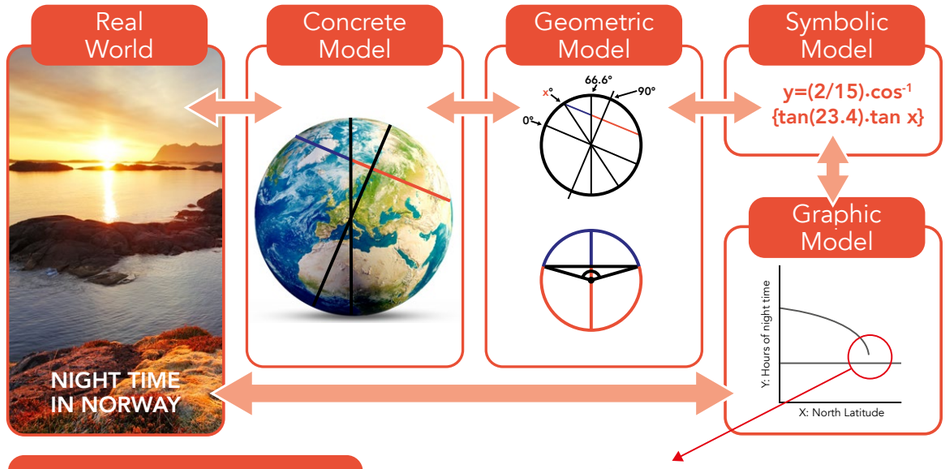
The second category has students adopting some social perspective; however, their responses are not specific. For instance, they will acknowledge that mathematics is useful in society but not mention how it is useful or provide examples.

In the third category, students assume a societal viewpoint and refer to a specific context, such as how mathematics can help a particular company reduce its manufacturing costs, but they do not include a general context.



Experimental teaching: the number of years to double their savings.

Mathematical Modelling Ex. Night time in Norway



An example of using mathematics to explain and predict phenomena.

The fourth category has students adopting a societal perspective and referencing a general context, such as using mathematics to explain phenomena, contribute to decision making, or assist with designing objects.

INVESTIGATION

Professor Ikeda conducted an experimental program with a ninth-grade class of approximately 30 students. The procedure was carried out over two years (2007–2008) at a junior high school in Japan with the same teacher during both years. A teaching program comprising nine lessons was devised. These lessons took place once a week from September to November each year and were carried out by the classroom teacher. The first eight lessons each lasted 100 minutes and the ninth lesson took 50 minutes.

THE EXPERIMENTAL TEACHING PROGRAM

The program of lessons concentrated on three roles of mathematics in society: understanding, making decisions, and designing. Eight modelling tasks were selected. The 'understanding' tasks involved investigating reflections of a face in a mirror and calculating probabilities in rock-paper-scissors. During the 'making decisions' tasks, students examined a bank interest system, calculated the number of years to double their savings and explored the position of tennis serves. The 'designing' tasks included investigating the shapes of cans, exploring the structure of a bicycle reflector and designing a parking space.

Having students recognise the roles that mathematics takes in society is a significant aim in the teaching and learning of mathematical modelling.

The students were asked to write down their responses to the pre-program question: 'How is mathematics useful when we examine real-world problems from various perspectives?' before commencing the program of nine lessons.

During the first seven lessons, the teacher conducted group discussions regarding the modelling process. These were followed by problem-solving periods where the teacher presented one of the modelling problems. The students worked on the problem in groups of 4 or 5 and

TO MAKE MONEY DOUBLE

X	1	2	3	4	5	6	7	8	9	10
Y	70	36	24	18	15	12	11	10	9	8
a	70	72	72	72	75	72	77	80	81	80

VALUE OF A

Calculation of average: $a = 75.1$

$y = 75.1/x$

A simple model to calculate the number of years to double their savings.

then came together to discuss it as a class. In the seventh and eighth lessons, three tasks were introduced and each group of students selected one to work on.

Throughout the lessons, emphasis was put on the reason for solving problems and students were encouraged to reflect on the roles of mathematics in order to identify the significant aspects of the roles of mathematics in society. The students first reflected prior to the teaching program; followed by reflection after each lesson; and finally, after the teaching program was completed.

During the ninth lesson, the students reviewed the eight modelling tasks. They reflected on the modelling processes and identified the common points in order to ascertain the roles of mathematics. The students discussed the problems they faced during the eight tasks and how they solved them. The students then broke into groups to the post-program question: 'How is mathematics useful when we examine real-world problems from various perspectives?'. Then the teacher asked each of the students to write down their answer without any teacher-led discussion so that the students could provide their own answers supported by their group discussion, but without any influence from the teacher.

Experimental teaching: investigating reflections of a face in a mirror.

ANALYSIS
A total of 57 students took part in the study, 31 students during the first year and 26 students during the second. The students' responses to the pre- and post-program questions regarding their perception of the roles of mathematics in society were coded and then analysed. Ikeda's analytical tool revealed that students' opinions concerning the roles of mathematics in society changed significantly over the course of the nine-week experimental teaching program.

Analysis of the students' perception relating to personal-societal perspectives, clarity of role statements, and specific-general contexts was carried out. A few students continued to adhere only to their personal perspectives or provide vague statements, insisting that mathematics was not useful in their lives, even though they acknowledged that it might be useful for others.

With respect to the personal-societal perspective, only a few students adopted a personal perspective without also developing a societal perspective. Regarding the clarity of role statements, several students still only used vague, general statements. From the specific-general contexts, the analytical tool revealed that some students were able to clearly identify several purposes of mathematical modelling.

Professor Ikeda found that by using the analytical tool he could start to distinguish between the qualitative differences of the various students' perceptions of the roles of mathematics in society as well as whether students were able to appreciate the roles from both personal and societal perspectives or only a personal perspective. He also discovered that students, who were able to perceive the roles of mathematics in society, did not automatically appreciate the utility of mathematics.

Students' opinions concerning the roles of mathematics in society changed significantly over the course of the nine-week experimental teaching program.

CONCLUSION
Professor Ikeda found strong evidence to suggest that this type of teaching program is significantly effective in developing students' thinking and their appreciation of mathematical modelling. This was reinforced when he examined the work that students produced in the study, demonstrating their thoughts on solving the set tasks. Ikeda found that these written examples of students' thinking consistently demonstrated a marked increase in the quality and depth of the students' insight into the mathematical modelling process.

This study also revealed that Professor Ikeda's analytical tool enabled the clarification of students' perceptions of the roles of mathematics in society, both before and after the teaching program was performed. This revealed how students' perceptions regarding the roles and utilisation of mathematics in society changed significantly over the course of the nine-week teaching program.

Professor Ikeda recommends that further attention should be paid to both the validity and reliability of his analytical tool, particularly when setting up the pre- and post-program question and interpreting of the students' response. He suggests that samples of students are interviewed to check the validity of the results. Keeping these points in mind, this analytical model still merits further use in future studies, particularly those involving student belief regarding the value of mathematics in society.

Behind the Research

Professor Toshikazu Ikeda

E: toshi@ynu.ac.jp **T:** +81 45 339 3371 **W:** http://er-web.jmk.ynu.ac.jp/html/IKEDA_Toshikazu/en.html
W: www.researchgate.net/profile/Toshikazu_Ikeda **W:** https://slideplayer.com/slide/8309101/
W: https://link.springer.com/article/10.1007/s11858-018-0927-3

Research Objectives

Toshikazu Ikeda, Professor of Mathematics Education at Yokohama National University, work focuses on teaching mathematical modelling and its applications. He has recently developed an analytical tool to evaluate the changes in students' perceptions of the roles of mathematics in society following an experimental teaching program.

Detail

Yokohama National University
40-8501 Kanagawa Prefecture,
Yokohama, Hodogaya-ku,
Japan

Bio
Toshikazu Ikeda is a Professor of Mathematics Education at Yokohama National University. He received his PhD in education at Waseda University in 2014. Since 1999, he has studied the teaching of mathematical modelling and applications, funded by a Grant-in-aid for Science Research in Japan.

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YNU 横浜国立大学 YOKOHAMA National University
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Personal Response

What initially prompted you to investigate students' perceptions of the roles of mathematics in society?

" In Japan, there has been the tendency where students have not realised the usefulness of mathematics in society from the result of PISA. To overcome this tendency, mathematical modelling has been gradually emphasised in the teaching of mathematics. However, I am uncertain that students will realise the usefulness of mathematics by focusing on mathematical modelling in mathematics lessons. If students thought that mathematics might not be used in his/her future life, he/she might not realise the usefulness of mathematics even though he/she realises that mathematics is applied in the real world. This study starts from this question. "