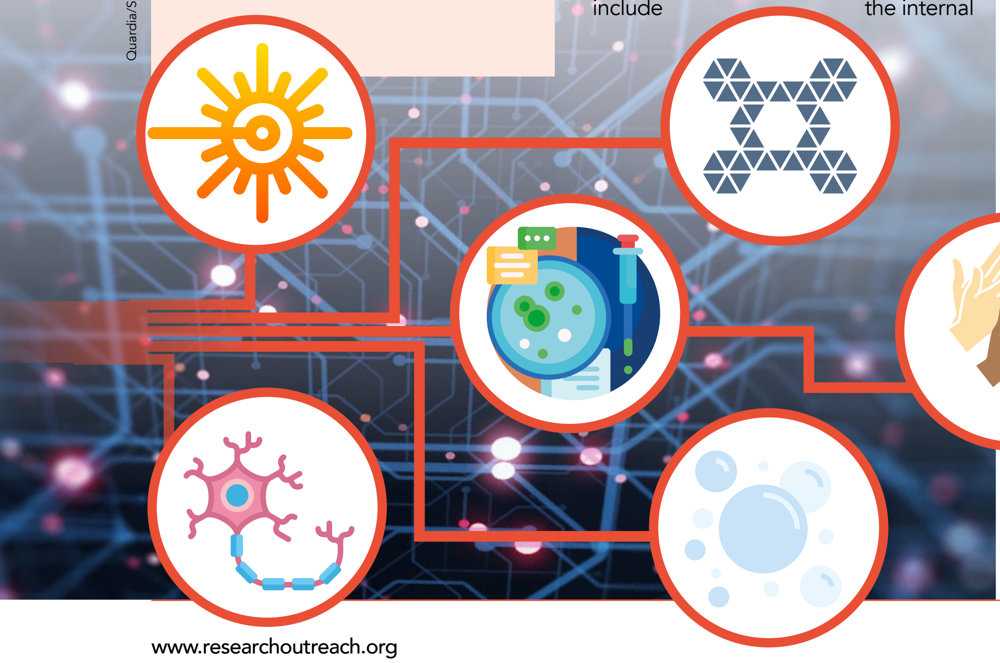
Synchronisation in symmetrically and asymmetrically coupled oscillators

Complex networks are systems composed of several components which may interact with each other. Examples range from laser arrays to the human brain, global climate, and digital communication forms. The behaviour of complex systems depends upon the internal dynamics of the individual unit, communication among cells, type of coupling between them, and symmetry. Dr Antonio Palacios from San Diego State University, San Diego, CA, studies synchronisation states in symmetrically and asymmetrically coupled oscillators.



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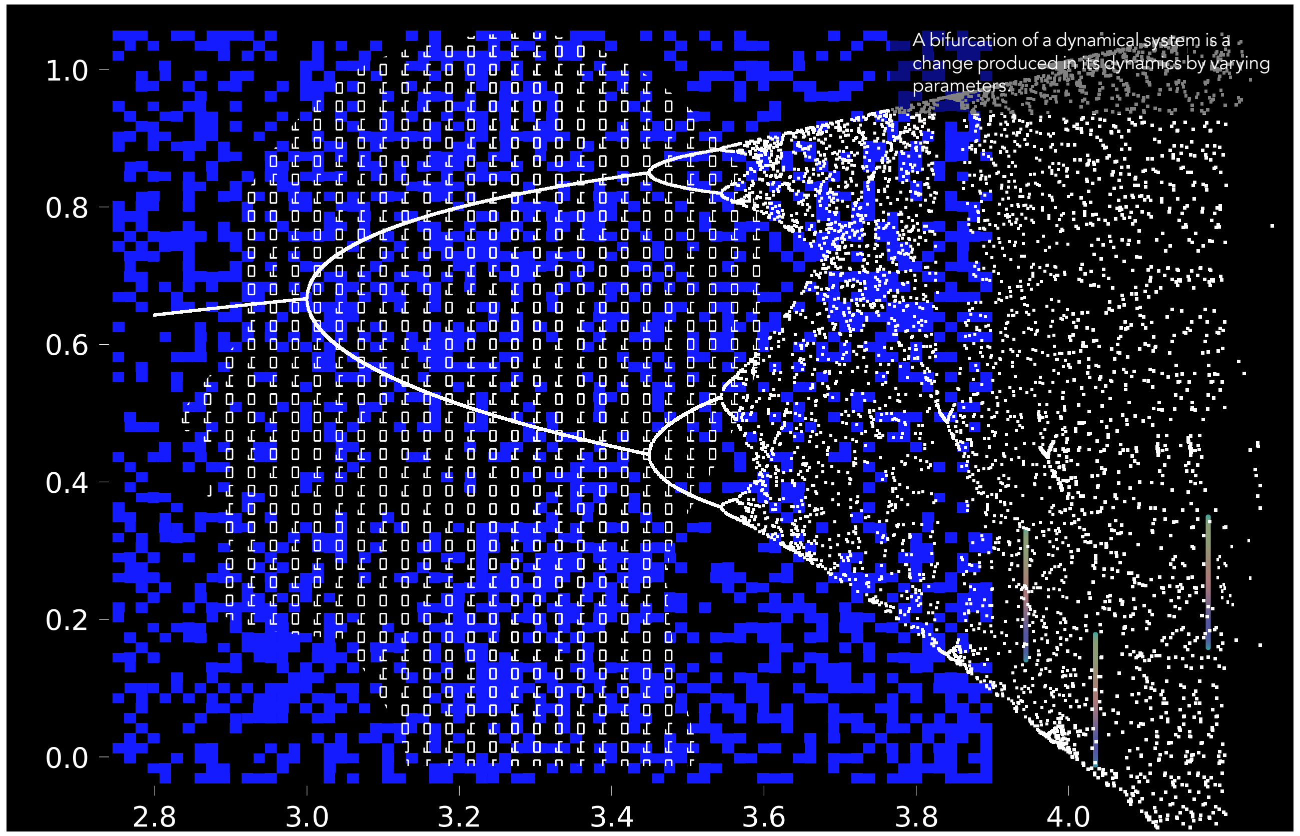
Complex networks are a great example of collective behaviour: the individual units or cells do not behave in the same way the whole network does. Studies into complex networks have gained great interest in past decades because of the development of interdisciplinary methods and theoretical models which can analyse the collective behaviour of the network. These networks include everything from laser arrays to digital communication, microbiology, neuroanatomy, Josephson junctions, central pattern generators in biological systems, coupled laser systems, chaotic oscillators, the collective behaviour of bubbles in fluidisation, the flocking of birds, and even psychology. The factors affecting the collective behaviour of complex networks include the internal dynamics of each unit, communication among the cells, and the type of coupling between them.

More recently, a fourth factor is also studied: symmetry. Symmetry is the property of 'appearing the same from different viewpoints'. It has been found that symmetry can usefully explain certain features in the collective behaviour of a complex network. One of the features of great interest is 'synchronisation', meaning the coordination of events to operate a system in unison. Typically, synchronisation states are found in identical units.

Recent findings (Nishikawa & Motter, 2016) favour the idea of asymmetry-induced symmetry. The authors have explained this by using a model system consisting of a network of oscillators that can oscillate in x and y directions only. The oscillators are identical if they have the same parameters and are non-identical for different parameters. The oscillators are symmetrically coupled if, at a given time, they all move in the same direction and asymmetric otherwise. The researchers have found that with asymmetrically coupled oscillators, synchronisation is stable only when the oscillators are non-identical.



A bifurcation of a dynamical system is a change produced in its dynamics by varying parameters.

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Dr Antonio Palacios from San Diego University, CA, contradicts these results. Using ideas and methods from perturbation analysis and equivariant bifurcation theory, he has proved the existence and stability of synchronisation in asymmetrically coupled identical oscillators. Perturbation analysis is a class of analytical methods for determining approximate solutions of nonlinear equations for which exact solutions cannot be obtained. A bifurcation of a dynamical system is a change produced in its dynamics by varying parameters.

THE IDEA OF SYMMETRY

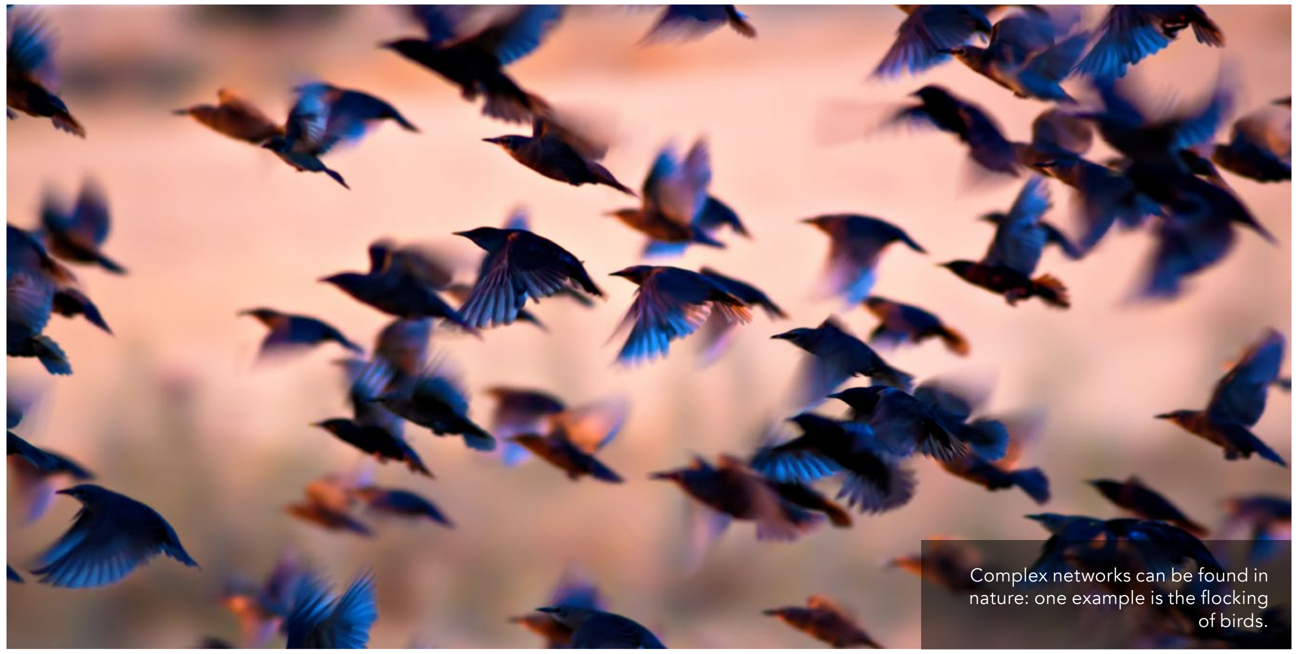
The idea of symmetry is fascinating. Indeed, it is often believed that physics is the study of symmetry, with theoretical physicist Hermann Weyl suggesting that 'all a priori statements in physics have their origin in symmetry'. However, for a long time, the idea of symmetry was not used for real complex systems since it was believed that they could not show symmetries. It has now been found, however, that real complex systems do show numerous symmetries which affect their dynamic behaviours (behaviour that continuously changes according to some conditions).

A crowd clapping in unison, crickets singing in synchrony, and fireflies blinking together are all examples of synchronisation in daily life. Synchronisation is a key parameter to our existence. The neurons in the brain working synchronously to generate signals and synchronic beating of the heart are some of the examples. The concept of spontaneous synchronisation has been long used for studying the behavioural uniformity that can emerge from interactions in complex systems such as crowd clapping in accord. There is complete synchronisation in the entire network and the constituent units also inherit the system symmetry.

Symmetry can usefully explain certain features in the collective behaviour of a complex network.

However, in some systems, the symmetry is lowered (symmetry breaking), and for such systems, the symmetric states are not always stable. The behaviour of real dynamical systems, such as in an all-to-all network, has been recently investigated. In parallel computing, all-to-all is a collective operation, where each processor sends an individual message to every other processor. Both synchronous and non-synchronous behaviour are observed in them. Their behaviour results from symmetry breaking since the realised state is less symmetric than the original system.

Synchronisation dynamics can be found in various fields around us, such as physics and engineering, biology, and social sciences. Two types of twin processes are associated with it: consensus dynamics and convergence to uniform patterns. In consensus dynamics, multi-agent systems achieve common goals by exchange of information via physiological systems, gene networks, large-scale energy systems, etc. The collective opinion formation in social networks is due to consensus



Complex networks can be found in nature: one example is the flocking of birds.

dynamics. In the case of convergence to uniform patterns, as time passes the system loses its initial distribution. Convergence to equilibrium in chaotic chemical reaction systems and population dispersion in natural systems involve convergence to uniformity.

Nishikawa and Motter (2016) have demonstrated asymmetry-induced symmetry by reversing the role of the system and its state. They assumed the constituent state to be asymmetric and the system as symmetric. They have discovered that complete synchronisation is not stable for identically coupled identical oscillators but becomes stable when the oscillators are non-identical. Hence the system breaks to preserve the state symmetry. Nishikawa and Motter give the mechanism for the formation of uniform patterns from the non-uniform ones. This is particularly helpful in understanding the development of five-fold radial symmetry in adult starfish from bilateral symmetry in starfish larvae, development of spherical symmetry in yeast cells from asymmetric bud cells, and recovery of lost symmetry in severed animals via regeneration.

CONTRADICTION TO THE IDEA OF 'SYMMETRIC STATES REQUIRING SYSTEM ASYMMETRY'

The synchronisation states are found in symmetrically coupled networks, with identical units. Dr Palacios contradicts the claims of Nishikawa and Motter that in networks with asymmetrically coupled oscillators, synchronisation can only be found to be stable when the oscillators are non-identical. Dr Palacios has proved them wrong first from the mathematical standpoint.

Dr Palacios proved that stable synchronisation states can, and do, occur in asymmetrically coupled networks of homogeneous oscillators.

According to his claims, the existence of stable synchronisation in networks of asymmetrically coupled non-identical oscillators does not necessarily exclude the existence of the same type of behaviour with identical oscillators. He further proved, both analytically and computationally, that stable synchronisation states can, and do, occur in asymmetrically coupled networks of homogeneous oscillators.

The simulations are used for a network of seven oscillators, which presents the emergence of unstable synchronisation in a network with identical oscillators, while stable synchronisation is shown to appear with non-identical oscillators. Further, it is revealed that the same network can also support stable synchronisation with homogeneous oscillators, contradicting the findings of Nishikawa and Motter.

Dr Palacios proved his results, for networks of arbitrary size, using ideas from perturbation analysis and equivariant bifurcation theory. He used perturbation theory to understand the source of instability of the synchronisation state. Given bifurcation theory in systems with symmetry, solutions of bifurcation problems lose symmetry as some parameters are varied, even though the equations retain the full symmetry of the system, i.e., there exists spontaneous symmetry breaking. However, the bifurcation equations can also possess less symmetry when some parameters are non-zero, which is known as forced symmetry breaking. The existence and stability of the synchronisation state for both types of networks, with identical and non-identical oscillators, is determined by Hopf bifurcation. The conclusions made by Dr Palacios will lead to a better understanding of the role of symmetry in complex networks.

Behind the Research

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Research Objectives

Dr Palacios studies synchronisation in symmetrically and asymmetrically coupled oscillators.

Detail

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Bio

Antonio Palacios received his PhD in Mathematics at Arizona State University. He was a postdoctoral fellow in Physics at the University of Houston with Professor Michael Gorman and then in Mathematics with Professor Marty Golubitsky. He joined the Math Department at San Diego State University in 1999. His research interests are in nonlinear systems.

Collaborators

Dr Palacios would like to acknowledge constructive discussions with Prof Pietro-Luciano Buono from Universite du Quebec at Rimouski.

San Diego State University

References

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Personal Response

How can the conclusions obtained from your research be helpful in the understanding of neuroanatomy and microbiology?

"The human brain contains approximately 10^{11} neurons. Many of these neurons form clusters, which can be associated with the generation and control of many neuro-physiological functions, e.g., mastication, breathing, gait patterns. In many instances, the collective patterns of activity of these neurons is the synchronisation state. Consequently, investigating the conditions for the existence and stability of synchronisation, in identical and non-identical clusters of neurons, is of great importance towards understanding how the brain, and the many neuro-physiological functions produced by it, function. The conclusions reached in this research manuscript might be able to help advance our understanding of these functions."